The Final Report of the National Mathematics Advisory Panel

2008

U.S. Department of Education

This report was produced under U.S. Department of Education Contract No. ED04CO0082/0001 for Widmeyer Communications and No. ED04CO0015/0006 for Abt Associates Inc. Ida Eblinger Kelley served as the contracting officer's technical representative. The views expressed herein do not necessarily represent the positions or policies of the Department of Education. No official endorsement by the U.S. Department of Education of any product, commodity, service or enterprise mentioned in this publication is intended or should be inferred.

National Mathematics Advisory Panel
Tyrrell Flawn
Executive Director

March 2008

To order copies of this report,

write to: ED Pubs, Education Publications Center, U.S. Department of Education, P.O. Box 1398, Jessup, MD 20794-1398

or **fax** your request to: 1-301-470-1244

or **email** your request to: edpubs@inet.ed.gov

or **call** in your request toll free: 1-877-433-7827 (1-877-4-ED-PUBS). If 877 service is not yet available in your area, call 1-800-872-5327 (1-800-USA-LEARN). Those who use a telecommunications device for the deaf (TDD) or a teletypewriter (TTY), should call 1-877-576-7734.

or **order online** at www.ed.gov/pubs/edpubs.html.

This report is also available on the Department's Web site at: www.ed.gov/MathPanel.

On request, this publication is available in alternate formats, such as Braille, large print, audiotape, or computer diskette. For more information, please contact the Department's Alternate Format Center at 1-202-260-0852 or 1-202-260-0818.

CONTENTS

Executive Summary

Background

The eminence, safety, and well-being of nations have been entwined for centuries with the ability of their people to deal with sophisticated quantitative ideas. Leading societies have commanded mathematical skills that have brought them advantages in medicine and health, in technology and commerce, in navigation and exploration, in defense and finance, and in the ability to understand past failures and to forecast future developments. History is full of examples.

During most of the 20th century, the United States possessed peerless mathematical prowess—not just as measured by the depth and number of the mathematical specialists who practiced here but also by the scale and quality of its engineering, science, and financial leadership, and even by the extent of mathematical education in its broad population. But without substantial and sustained changes to its educational system, the United States will relinquish its leadership in the 21st century. This report is about actions that must be taken to strengthen the American people in this central area of learning. Success matters to the nation at large. It matters, too, to individual students and their families, because it opens doors and creates opportunities.

> ***During most of the 20th century, the United States possessed peerless mathematical prowess****—not just as measured by the depth and number of the mathematical specialists who practiced here, but also by the scale and quality of its engineering, science, and financial leadership.…*

Much of the commentary on mathematics and science in the United States focuses on national economic competitiveness and the economic well-being of citizens and enterprises. There is reason enough for concern about these matters, but it is yet more fundamental to recognize that the safety of the nation and the quality of life—not just the prosperity of the nation—are at issue.

In the contemporary world, an educated technical workforce undergirds national leadership. Yet the United States faces a future in which there will be accelerating retirements affecting a large fraction of the current science and engineering workforce, even as the growth of job opportunities in this sector is expected to outpace job growth in the economy at large. These trends will place substantial stress on the nation's ability to sustain a workforce with adequate scale and quality. For many years, our country has imported a great volume of technical talent from abroad, but the dramatic success of economies overseas in the age of the Internet casts doubt on the viability of such a strategy in the future, because attractive employment for technical workers is developing in countries that have been supplying invaluable talent for U.S. employers. From 1990 to 2003, research and development expenditures in Asian countries other than Japan grew from an insignificant percentage to almost half of American R&D expenditures. There are consequences to a weakening of American independence and leadership in mathematics, the natural sciences, and engineering. We risk our ability to adapt to change. We risk technological surprise to our economic viability and to the foundations of our country's security. National policy must ensure the healthy development of a domestic technical workforce of adequate scale with top-level skills.

But the concerns of national policy relating to mathematics education go far beyond those in our society who will become scientists or engineers. The national workforce of future years will surely have to handle quantitative concepts more fully and more deftly than at present. So will the citizens and policy leaders who deal with the public interest in positions of civic leadership. Sound education in mathematics across the population is a national interest.

Success in mathematics education also is important for individual citizens, because it gives them college and career options, and it increases prospects for future income. A strong grounding in high school mathematics through Algebra II or higher correlates powerfully with access to college, graduation from college, and earning in the top quartile of income from employment. The value of such preparation promises to be even greater in the future. The National Science Board indicates that the growth of jobs in the mathematics-intensive science and engineering workforce is outpacing overall job growth by 3:1.

***International and domestic comparisons show that American students have** not been succeeding in the mathematical part of their education at anything like a level expected of an international leader.*

International and domestic comparisons show that American students have not been succeeding in the mathematical part of their education at anything like a level expected of an international leader. Particularly disturbing is the consistency of findings that American students achieve in mathematics at a mediocre level by comparison to peers worldwide. On our own "National Report Card"—the National Assessment of Educational Progress (NAEP)—there are positive trends of scores at Grades 4 and 8, which have just reached historic highs. This is a sign of significant progress. Yet other results from NAEP are less positive: 32% of our students are at or above the "proficient" level in Grade 8, but only 23% are proficient at Grade 12. Consistent with these findings is the vast and growing demand for remedial mathematics education among arriving students in four-year colleges and community colleges across the nation.

Moreover, there are large, persistent disparities in mathematics achievement related to race and income—disparities that are not only devastating for individuals and families but also project poorly for the nation's future, given the youthfulness and high growth rates of the largest minority populations.

Although our students encounter difficulties with many aspects of mathematics, many observers of educational policy see Algebra as a central concern.[1] The sharp falloff in mathematics achievement in the U.S. begins as students reach late middle school, where, for more and more students, algebra course work begins. Questions naturally arise about how students can be best prepared for entry into Algebra.

These are questions with consequences, for Algebra is a demonstrable gateway to later achievement. Students need it for any form of higher mathematics later in high school; moreover, research shows that completion of Algebra II correlates significantly with success in college and earnings from employment. In fact, students who complete Algebra II are more than twice as likely to graduate from college compared to students with less mathematical preparation. Among African-American and Hispanic students with mathematics preparation at least through Algebra II, the differences in college graduation rates versus the student population in general are half as large as the differences for students who do not complete Algebra II.

Students who complete Algebra II are more than *twice as likely to graduate from college compared to students with less mathematical preparation.*

For all of these considerations, the President created the National Mathematics Advisory Panel in April 2006, with the responsibilities of relying upon the "best available scientific evidence" and recommending ways "…to foster greater knowledge of and improved performance in mathematics among American students."

Principal Messages

This Panel, diverse in experience, expertise, and philosophy, agrees broadly that the delivery system in mathematics education—the system that translates mathematical knowledge into value and ability for the next generation—is broken and must be fixed. This is not a conclusion about any single element of the system. It is about how the many parts do not now work together to achieve a result worthy of this country's values and ambitions.

On the basis of its deliberation and research, the Panel can report that America has genuine opportunities for improvement in mathematics education. This report lays them out for action.

The essence of the Panel's message is *to put first things first*. There are six elements, expressed compactly here, but in greater detail later.

- The mathematics curriculum in Grades PreK–8 should be streamlined and should emphasize a well-defined set of the most critical topics in the early grades.

[1] The word "algebra" is capitalized when referring to a particular course or course sequence, such as Algebra I and II.

- Use should be made of what is clearly known from rigorous research about how children learn, especially by recognizing a) the advantages for children in having a strong start; b) the mutually reinforcing benefits of conceptual understanding, procedural fluency, and automatic (i.e., quick and effortless) recall of facts; and c) that effort, not just inherent talent, counts in mathematical achievement.

- Our citizens and their educational leadership should recognize mathematically knowledgeable classroom teachers as having a central role in mathematics education and should encourage rigorously evaluated initiatives for attracting and appropriately preparing prospective teachers, and for evaluating and retaining effective teachers.

- Instructional practice should be informed by high-quality research, when available, and by the best professional judgment and experience of accomplished classroom teachers. High-quality research does not support the contention that instruction should be either entirely "student centered" or "teacher directed." Research indicates that some forms of particular instructional practices can have a positive impact under specified conditions.

- NAEP and state assessments should be improved in quality and should carry increased emphasis on the most critical knowledge and skills leading to Algebra.

- The nation must continue to build capacity for more rigorous research in education so that it can inform policy and practice more effectively.

Positive results can be achieved in a reasonable time at accessible cost, but a consistent, wise, community-wide effort will be required. Education in the United States has many participants in many locales—teachers, students, and parents; state school officers, school board members, superintendents, and principals; curriculum developers, textbook writers, and textbook editors; those who develop assessment tools; those who prepare teachers and help them to continue their development; those who carry out relevant research; association leaders and government officials at the federal, state, and local levels. All carry responsibilities. All can be important to success.

The network of these many participants is linked through interacting national associations. A coordinated national approach toward improved mathematics education will require an annual forum of their leaders for at least a decade. The Panel recommends that the U.S. Secretary of Education take the lead in convening the forum initially, charge it to organize in a way that will sustain an effective effort, and request a brief annual report on the mutual agenda adopted for the year ahead.

The President asked the Panel to use the best available scientific research to advise on improvements in the mathematics education of the nation's children. Our consistent respect for sound research has been the main factor enabling the Panel's joint conclusions on so many matters, despite differences of perspective and philosophy. At the same time, we found no research or insufficient research relating to a great many matters of concern in educational policy and practice. In those areas, the Panel has been very limited in what it can report.

The Panel lays out many concrete steps that can be taken now toward significantly improved mathematics education, but it also views them only as a best start in a long process. This journey, like that of the post-Sputnik era, will require a commitment to "learning as we go along." The nation should recognize that there is much more to discover about how to achieve better results. Models of continuous improvement have proven themselves in many other areas, and they can work again for America in mathematics education.

The National Mathematics Advisory Panel

The President established the Panel via Executive Order 13398 (Appendix A), in which he also assigned responsibility to the U.S. Secretary of Education for appointment of members and for oversight of the Panel. While the presidential charge contains many explicit elements, there is a clear emphasis on the preparation of students for entry into, and success in, Algebra.

Over a period of 20 months, the Panel received public testimony as a committee of the whole but worked largely in task groups and subcommittees dedicated to major components of the presidential charge. Questions like the following illustrate the scope of the Panel's inquiry:

- What is the essential content of school algebra and what do children need to know before starting to study it?
- What is known from research about how children learn mathematics?
- What is known about the effectiveness of instructional practices and materials?
- How can we best recruit, prepare, and retain effective teachers of mathematics?
- How can we make assessments of mathematical knowledge more accurate and more useful?
- What do practicing teachers of algebra say about the preparation of students whom they receive into their classrooms and about other relevant matters?
- What are the appropriate standards of evidence for the Panel to use in drawing conclusions from the research base?

Each of five task groups carried out a detailed analysis of the available evidence in a major area of the Panel's responsibility: Conceptual Knowledge and Skills, Learning Processes, Instructional Practices, Teachers and Teacher Education, and Assessment. Each of three subcommittees was charged with completion of a particular advisory function for the Panel: Standards of Evidence, Instructional Materials, and the Panel-commissioned National Survey of Algebra Teachers (see sidebar, page 9). Each task group and subcommittee produced a report supporting this document. All eight reports are separately available.

The Panel took consistent note of the President's emphasis on "the best available scientific evidence" and set a high bar for admitting research results into consideration. In essence, the Panel required the work to have been carried out in a way that manifested rigor and could support generalization at the level of significance to policy. One of the subcommittee reports covers global considerations relating to standards of evidence, while individual task group reports amplify the standards in the particular context of each task group's work. In all, the Panel reviewed more than 16,000 research publications and policy reports and received public testimony from 110 individuals, of whom 69 appeared before the Panel on their own and 41 others were invited on the basis of expertise to cover particular topics. In addition, the Panel reviewed written commentary from 160 organizations and individuals, and analyzed survey results from 743 active teachers of algebra.

***The Panel took consistent note of the President's** emphasis on "the best available scientific evidence" and set a high bar for admitting research results into consideration.*

In late 2007, the Panel synthesized this Final Report by drawing together the most important findings and recommendations, which are hereby issued with the Panel's full voice. This report connects in many places to the eight reports of the task groups and subcommittees, which carry detailed analyses of research literature and other relevant materials. These supporting reports cover work carried out as part of the Panel's overall mission, but they are presented by only those members who participated in creating them. This Final Report represents findings and recommendations of the Panel as a whole.

Main Findings and Recommendations

The Panel had a broad scope and reached many individual findings and recommendations, all conveyed in the main report under headings corresponding to those below. This Executive Summary generally contains only abbreviated versions of the most important points.

Curricular Content

1) A focused, coherent progression of mathematics learning, with an emphasis on proficiency with key topics, should become the norm in elementary and middle school mathematics curricula. Any approach that continually revisits topics year after year without closure is to be avoided.

By the term *focused*, the Panel means that curriculum must include (and engage with adequate depth) the most important topics underlying success in school algebra. By the term *coherent*, the Panel means that the curriculum is marked by effective, logical progressions from earlier, less sophisticated topics into later, more sophisticated ones. Improvements like those suggested in this report promise immediate positive results with minimal additional cost.

By the term *proficiency*, the Panel means that students should understand key concepts, achieve automaticity as appropriate (e.g., with addition and related subtraction facts), develop flexible, accurate, and automatic execution of the standard algorithms, and use these competencies to solve problems.[2]

2) To clarify instructional needs in Grades PreK–8 and to sharpen future discussion about the role of school algebra in the overall mathematics curriculum, the Panel developed a clear concept of school algebra via its list of Major Topics of School Algebra (Table 1, page 16).

   School algebra is a term chosen to encompass the full body of algebraic material that the Panel expects to be covered through high school, regardless of its organization into courses and levels. The Panel expects students to be able to proceed successfully at least through the content of Algebra II.

3) The Major Topics of School Algebra in Figure 1 should be the focus for school algebra standards in curriculum frameworks, algebra courses, textbooks for algebra, and in end-of-course assessments.

4) A major goal for K–8 mathematics education should be proficiency with fractions (including decimals, percents, and negative fractions), for such proficiency is foundational for algebra and, at the present time, seems to be severely underdeveloped. Proficiency with whole numbers is a necessary precursor for the study of fractions, as are aspects of measurement and geometry. These three areas—whole numbers, fractions, and particular aspects of geometry and measurement—are the Critical Foundations of Algebra. Important elements within each of these three categories are delineated on page 17 of this report.

[2] This meaning is in keeping with *Adding It Up* (National Research Council, 2001, p. 116), in which five attributes were associated with the concept of proficiency: 1) conceptual understanding (comprehension of mathematical concepts, operations, and relations), 2) procedural fluency (skills in carrying out procedures flexibly, fluently, and appropriately), 3) strategic competence (ability to formulate, represent, and solve mathematical problems), 4) adaptive reasoning (capacity for logical thought, reflection, explanation, and justification), and 5) productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

The Critical Foundations are not meant to comprise a complete mathematics curriculum leading to algebra; however, they deserve primary attention and ample time in any mathematics curriculum.

5) To encourage the development of students in Grades PreK–8 at an effective pace, the Panel recommends a set of Benchmarks for the Critical Foundations (Table 2, page 20). They should be used to guide classroom curricula, mathematics instruction, textbook development, and state assessments.

6) All school districts should ensure that all prepared students have access to an authentic algebra course—and should prepare more students than at present to enroll in such a course by Grade 8. The word *authentic* is used here as a descriptor of a course that addresses algebra consistently with the Major Topics of School Algebra (Table 1, page 16). Students must be prepared with the mathematical prerequisites for this course according to the Critical Foundations of Algebra (page 17) and the Benchmarks for the Critical Foundations (Table 2, page 20).

7) Teacher education programs and licensure tests for early childhood teachers, including all special education teachers at this level, should fully address the topics on whole numbers, fractions, and the appropriate geometry and measurement topics in the Critical Foundations of Algebra, as well as the concepts and skills leading to them; for elementary teachers, including elementary level special education teachers, all topics in the Critical Foundations of Algebra and those topics typically covered in an introductory Algebra course; and for middle school teachers, including middle school special education teachers, the Critical Foundations of Algebra and all of the Major Topics of School Algebra.

Learning Processes

8) Most children acquire considerable knowledge of numbers and other aspects of mathematics before they enter kindergarten. This is important, because the mathematical knowledge that kindergartners bring to school is related to their mathematics learning for years thereafter—in elementary school, middle school, and even high school. Unfortunately, most children from low-income backgrounds enter school with far less knowledge than peers from middle-income backgrounds, and the achievement gap in mathematical knowledge progressively widens throughout their PreK–12 years.

9) Fortunately, encouraging results have been obtained for a variety of instructional programs developed to improve the mathematical knowledge of preschoolers and kindergartners, especially those from low-income backgrounds. There are effective techniques—derived from scientific research on learning—that could be put to work in the classroom today to improve children's mathematical knowledge.

However, tests of both short-term and long-term effects of these interventions with larger populations of children from low-income families are urgently needed.

10) To prepare students for Algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency, and problem-solving skills. Debates regarding the relative importance of these aspects of mathematical knowledge are misguided. These capabilities are mutually supportive, each facilitating learning of the others. Teachers should emphasize these interrelations; taken together, conceptual understanding of mathematical operations, fluent execution of procedures, and fast access to number combinations jointly support effective and efficient problem solving.

11) Computational proficiency with whole number operations is dependent on sufficient and appropriate practice to develop automatic recall of addition and related subtraction facts, and of multiplication and related division facts. It also requires fluency with the standard algorithms for addition, subtraction, multiplication, and division. Additionally it requires a solid understanding of core concepts, such as the commutative, distributive, and associative properties. Although the learning of concepts and algorithms reinforce one another, each is also dependent on different types of experiences, including practice.

12) Difficulty with fractions (including decimals and percents) is pervasive and is a major obstacle to further progress in mathematics, including algebra. A nationally representative sample of teachers of Algebra I who were surveyed for the Panel rated students as having very poor preparation in "rational numbers and operations involving fractions and decimals."

    As with learning whole numbers, a conceptual understanding of fractions and decimals and the operational procedures for using them are mutually reinforcing. One key mechanism linking conceptual and procedural knowledge is the ability to represent fractions on a number line. The curriculum should afford sufficient time on task to ensure acquisition of conceptual and procedural knowledge of fractions and of proportional reasoning. Instruction focusing on conceptual knowledge of fractions is likely to have the broadest and largest impact on problem-solving performance when it is directed toward the accurate solution of specific problems.

13) Mathematics performance and learning of groups that have traditionally been underrepresented in mathematics fields can be improved by interventions that address social, affective, and motivational factors. Recent research documents that social and intellectual support from peers and teachers is associated with higher mathematics performance for all students, and that such support is especially important for many African-American and Hispanic students. There is an urgent need to conduct experimental evaluations of the effectiveness of support-focused

interventions both small- and large-scale, because they are promising means for reducing the mathematics achievement gaps that are prevalent in U.S. society.

14) Children's goals and beliefs about learning are related to their mathematics performance. Experimental studies have demonstrated that changing children's beliefs from a focus on ability to a focus on effort increases their engagement in mathematics learning, which in turn improves mathematics outcomes: When children believe that their efforts to learn make them "smarter," they show greater persistence in mathematics learning. Related research demonstrates that the engagement and sense of efficacy of African-American and Hispanic students in mathematical learning contexts not only tends to be lower than that of white and Asian students but also that it can be significantly increased.

    Teachers and other educational leaders should consistently help students and parents to understand that an increased emphasis on the importance of effort is related to improved mathematics performance. This is a critical point because much of the public's self-evident resignation about mathematics education (together with the common tendencies to dismiss weak achievement and to give up early) seems rooted in the erroneous idea that success is largely a matter of inherent talent or ability, not effort.

15) Teachers and developers of instructional materials sometimes assume that students need to be a certain age to learn certain mathematical ideas. However, a major research finding is that what is developmentally appropriate is largely contingent on prior opportunities to learn. Claims based on theories that children of particular ages cannot learn certain content because they are "too young," "not in the appropriate stage," or "not ready" have consistently been shown to be wrong. Nor are claims justified that children cannot learn particular ideas because their brains are insufficiently developed, even if they possess the prerequisite knowledge for learning the ideas.

Teachers and Teacher Education

16) Teachers who consistently produce significant gains in students' mathematics achievement can be identified using value-added analyses (analyses that examine individual students' achievement gains as a function of the teacher). The impact on students' mathematics learning is compounded if students have a series of these more effective teachers.

Unfortunately, little is known from existing high-quality research about what effective teachers do to generate greater gains in student learning. Further research is needed to identify and more carefully define the skills and practices underlying these differences in teachers' effectiveness, and how to develop them in teacher preparation programs.

17) Research on the relationship between teachers' mathematical knowledge and students' achievement confirms the importance of teachers' content knowledge. It is self-evident that teachers cannot teach what they do not know. However, because most studies have relied on proxies for teachers' mathematical knowledge (such as teacher certification or courses taken), existing research does not reveal the specific mathematical knowledge and instructional skill needed for effective teaching, especially at the elementary and middle school level. Direct assessments of teachers' actual mathematical knowledge provide the strongest indication of a relation between teachers' content knowledge and their students' achievement. More precise measures are needed to specify in greater detail the relationship among elementary and middle school teachers' mathematical knowledge, their instructional skill, and students' learning.

18) Teaching well requires substantial knowledge and skill. However, existing research on aspects of teacher education, including standard teacher preparation programs, alternative pathways into teaching, support programs for new teachers (e.g., mentoring), and professional development, is not of sufficient rigor or quality to permit the Panel to draw conclusions about the features of professional training that have effects on teachers' knowledge, their instructional practice, or their students' achievement.

    Currently there are multiple pathways into teaching. Research indicates that differences in teachers' knowledge and effectiveness between these pathways are small or nonsignificant compared to very large differences among the performance of teachers within each pathway.

19) The mathematics preparation of elementary and middle school teachers must be strengthened as one means for improving teachers' effectiveness in the classroom. This includes preservice teacher education, early career support, and professional development programs. A critical component of this recommendation is that teachers be given ample opportunities to learn mathematics for teaching. That is, teachers must know in detail and from a more advanced perspective the mathematical content they are responsible for teaching and the connections of that content to other important mathematics, both prior to and beyond the level they are assigned to teach.

High-quality research must be undertaken to create a sound basis for the mathematics preparation of elementary and middle school teachers within preservice teacher education, early-career support, and ongoing professional development programs. Outcomes of different approaches should be evaluated by using reliable and valid measures of their effects on prospective and current teachers' instructional techniques and, most importantly, their effects on student achievement.

20) In an attempt to improve mathematics learning at the elementary level, a number of school districts around the country are using "math specialist teachers" of three different types—math coaches (lead teachers), full-time elementary mathematics teachers, and pull-out teachers. However, the Panel found no high-quality research showing that the use of any of these types of math specialist teachers improves students' learning.

The Panel recommends that research be conducted on the use of full-time mathematics teachers in elementary schools. These would be teachers with strong knowledge of mathematics who would teach mathematics full-time to several classrooms of students, rather than teaching many subjects to one class, as is typical in most elementary classrooms. This recommendation for research is based on the Panel's findings about the importance of teachers' mathematical knowledge. The use of teachers who have specialized in elementary mathematics teaching could be a practical alternative to increasing all elementary teachers' content knowledge (a problem of huge scale) by focusing the need for expertise on fewer teachers.

21) Schools and teacher education programs should develop or draw on a variety of carefully evaluated methods to attract and prepare teacher candidates who are mathematically knowledgeable and to equip them with the skills to help students learn mathematics.

22) Research on teacher incentives generally supports their effectiveness, although the quality of the studies is mixed. Given the substantial number of unknowns, policy initiatives involving teacher incentives should be carefully evaluated.

Instructional Practices

23) All-encompassing recommendations that instruction should be entirely "student centered" or "teacher directed" are not supported by research. If such recommendations exist, they should be rescinded. If they are being considered, they should be avoided. High-quality research does not support the exclusive use of either approach.

24) Research has been conducted on a variety of cooperative learning approaches. One such approach, Team Assisted Individualization (TAI), has been shown to improve students' computation skills. This highly structured pedagogical strategy involves heterogeneous groups of students helping

each other, individualized problems based on student performance on a diagnostic test, specific teacher guidance, and rewards based on both group and individual performance. Effects of TAI on conceptual understanding and problem solving were not significant.

25) Teachers' regular use of formative assessment improves their students' learning, especially if teachers have additional guidance on using the assessment to design and to individualize instruction. Although research to date has only involved one type of formative assessment (that based on items sampled from the major curriculum objectives for the year, based on state standards), the results are sufficiently promising that the Panel recommends regular use of formative assessment for students in the elementary grades.

26) The use of "real-world" contexts to introduce mathematical ideas has been advocated, with the term "real world" being used in varied ways. A synthesis of findings from a small number of high-quality studies indicates that if mathematical ideas are taught using "real-world" contexts, then students' performance on assessments involving similar "real-world" problems is improved. However, performance on assessments more focused on other aspects of mathematics learning, such as computation, simple word problems, and equation solving, is not improved.

27) Explicit instruction with students who have mathematical difficulties has shown consistently positive effects on performance with word problems and computation. Results are consistent for students with learning disabilities, as well as other students who perform in the lowest third of a typical class. By the term *explicit instruction*, the Panel means that teachers provide clear models for solving a problem type using an array of examples, that students receive extensive practice in use of newly learned strategies and skills, that students are provided with opportunities to think aloud (i.e., talk through the decisions they make and the steps they take), and that students are provided with extensive feedback.

    This finding does not mean that all of a student's mathematics instruction should be delivered in an explicit fashion. However, the Panel recommends that struggling students receive some explicit mathematics instruction regularly. Some of this time should be dedicated to ensuring that these students possess the foundational skills and conceptual knowledge necessary for understanding the mathematics they are learning at their grade level.

28) Research on instructional software has generally shown positive effects on students' achievement in mathematics as compared with instruction that does not incorporate such technologies. These studies show that technology-based drill and practice and tutorials can improve student performance in specific areas of mathematics. Other studies show that teaching computer programming to students can support the development of particular mathematical concepts, applications, and problem solving.

However, the nature and strength of the results vary widely across these studies. In particular, one recent large, multisite national study found no significant effects of instructional tutorial (or tutorial and practice) software when implemented under typical conditions of use. Taken together, the available research is insufficient for identifying the factors that influence the effectiveness of instructional software under conventional circumstances.

29) A review of 11 studies that met the Panel's rigorous criteria (only one study less than 20 years old) found limited or no impact of calculators on calculation skills, problem solving, or conceptual development over periods of up to one year. This finding is limited to the effect of calculators as used in the 11 studies. However, the Panel's survey of the nation's algebra teachers indicated that the use of calculators in prior grades was one of their concerns. The Panel cautions that to the degree that calculators impede the development of automaticity, fluency in computation will be adversely affected.

    The Panel recommends that high-quality research on particular uses of calculators be pursued, including both their short- and long-term effects on computation, problem solving, and conceptual understanding.

30) Mathematically gifted students with sufficient motivation appear to be able to learn mathematics much faster than students proceeding through the curriculum at a normal pace, with no harm to their learning, and should be allowed to do so.

Instructional Materials

31) U.S. mathematics textbooks are extremely long—often 700–1,000 pages. Excessive length makes books more expensive and can contribute to a lack of coherence. Mathematics textbooks are much smaller in many nations with higher mathematics achievement than the U.S., thus demonstrating that the great length of our textbooks is not necessary for high achievement. Representatives of several publishing companies who testified before the Panel indicated that one substantial contributor to the length of the books was the demand of meeting varying state standards for what should be taught in each grade. Other major causes of the extreme length of U.S. mathematics textbooks include the many photographs, motivational stories, and other nonmathematical content that the books include. Publishers should make every effort to produce much shorter and more focused mathematics textbooks.

32) States and districts should strive for greater agreement regarding which topics will be emphasized and covered at particular grades. Textbook publishers should publish editions that include a clear emphasis on the material that these states and districts agree to teach in specific grades.

33) Publishers must ensure the mathematical accuracy of their materials. Those involved with developing mathematics textbooks and related instructional materials need to engage mathematicians, as well as mathematics educators, at all stages of writing, editing, and reviewing these materials.

Assessment

34) NAEP and state tests for students through Grade 8 should focus on and adequately represent the Panel's Critical Foundations of Algebra. Student achievement on this critical mathematics content should be reported and tracked over time.

35) The Panel suggests that the NAEP strand on "Number Properties and Operations" be expanded and divided into two parts. The former should include a focus on whole numbers, including whole number operations (i.e., addition, subtraction, multiplication, division), at Grade 4, and on all integers (negative and positive) at Grade 8. The second content area involving number should focus on fractions. At Grade 4, it should involve beginning work with fractions and decimals, including recognition, representation, and comparing and ordering. The coverage should be expanded to include operations with fractions, decimals, and percents at Grade 8. Similarly, the content of work with whole numbers and fractions on state tests should expand and cover these concepts and operations as they develop from year to year, particularly at Grades 5, 6, and 7, which are grade levels when the NAEP test is not offered.

36) The Panel recommends a more appropriate balance in how algebra is defined and assessed at both the Grade 4 and Grade 8 levels of the NAEP. The Panel strongly recommends that "algebra" problems involving patterns should be greatly reduced in these tests. The same consideration applies to state tests.

37) State tests and NAEP must be of the highest mathematical and technical quality. To this end, states and NAEP should develop procedures for item development, quality control, and oversight to ensure that test items reflect the best item-design features, are of the highest mathematical and psychometric quality, and measure what is intended, with non-construct-relevant sources of variance in performance minimized (i.e., with nonmathematical sources of influence on student performance minimized).

38) Calculators should not be used on test items designed to assess computational facility.

Research Policies and Mechanisms

39) It is essential to produce methodologically rigorous scientific research in crucial areas of national need, such as the teaching and learning of mathematics. Researchers, educators, state and federal policymakers, private foundations, and research agencies have made and can continue to make important contributions toward this goal. Specifically, more research is needed that identifies: 1) effective instructional practices, materials, and principles of instructional design, 2) mechanisms of learning, 3) ways to enhance teachers' effectiveness, including teacher education, that are directly tied to objective measures of student achievement, and 4) item and test features that improve the assessment of mathematical knowledge. Although the number of such studies has grown in recent years due to changes in policies and priorities at federal agencies, these studies are only beginning to yield findings, and their number remains comparatively small.

40) As in all fields of education, the large quantity of studies gathered in literature searches on important topics in mathematics education is reduced appreciably once contemporary criteria for rigor and generalizability are applied. Therefore, the Panel recommends that governmental agencies that fund research give priority not only to increasing the supply of research that addresses mathematics education but also to ensuring that such projects meet stringent methodological criteria, with an emphasis on the support of studies that incorporate randomized controlled designs (i.e., designs where students, classrooms, or schools are randomly assigned to conditions and studied under carefully controlled circumstances) or methodologically rigorous quasi-experimental designs. These studies must possess adequate statistical power, which will require substantial funding.

    Both smaller-scale experiments on the basic science of learning and larger-scale randomized experiments examining effective classroom practices are needed to ensure the coherent growth of research addressing important questions in mathematics education. Basic research on causal mechanisms of learning, as well as randomized trials, are essential, and, depending on their methodologies, both can be rigorous and relevant to educational practice. Basic research, in particular, is necessary to develop explicit predictions and to test hypotheses, which are underemphasized in current research on mathematics education.

41) Leaders of graduate programs in education and related fields should ensure attention to research design, analysis, and interpretation for teachers and those entering academic and educational leadership positions in order to increase the national capacity to conduct and utilize rigorous research.

42) New funding should be provided to establish support mechanisms for career shifts (K, or career development, awards from the National

Institutes of Health represent one example). Many accomplished researchers who study the basic components of mathematics learning are not directly engaged in relevant educational research. While this more basic kind of research is important both in its own right and as a crucial foundation for designing classroom-level learning projects, at least some of these investigators have the potential to make more directly relevant contributions to educational research. Consequently, providing incentives for them to change the emphasis of their research programs could enhance research capacity in the field.

43) Support should be provided to encourage the creation of cross-disciplinary research teams, including expertise in educational psychology, sociology, economics, cognitive development, mathematics, and mathematics education.

44) PreK–12 schools should be provided with incentives and resources to provide venues for, and encourage collaboration in, educational research.

45) Unnecessary barriers to research should be lowered. Although existing guidelines for the protection of human subjects must be fully respected, Institutional Review Board procedures should be streamlined for educational research that qualifies as being of low or minimal risk. The resolutions of the National Board for Education Sciences concerning making individual student data available to researchers with appropriate safeguards for confidentiality should be supported.